Evaluation of H-infinity Filter in Time Differential Localization Systems

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Abstract— **This paper evaluates the performances of H_∞ filter in differential time of arrival (TDoA) localization applications and compares the results with other filters such as extended Kalman filter (EKF) and unscented Kalman filter (UKF) in practical trials. The filters are compared in ideal as well as non-ideal conditions such as different positive and negative values of initial points, presence of erroneous data and excessive noise. The results show that, the H_∞ filter is sub-optimal in ideal conditions as the other filters outperform this filter, but once the initial points are badly selected or data are erroneous, the filter is more robust and accurate than the other variants. Considering the challenging conditions of the indoor environment, use of this filter in localization systems seems to be a good choice.**

Keywords— Localization, TDoA, EKF, UKF, H-infinity filter

I. INTRODUCTION

Kalman-based family of filters are very flexible, accurate and effective type of estimators which have been applied successfully in many projects especially in the localization applications in the last few decades [1]. Among these filters, the extended Kalman filter (EKF) is commonly used for nonlinear cases. This filter benefits from Jacobian linearization process that extracts the first moment of the Taylor series to define a linear matrix at the operation point of the non-linear model. As in this process, the higher orders of the Taylor series are ignored, this filter is known to be sub-optimal in highly non-linear conditions. One solution to this problem is second order EKF (SOEKF) which considers the second term in Taylor series yielding in higher accuracy of the filter with the cost of higher computation time and complexity [2]. Other heuristic methods such as unscented Kalman filter (UKF) can also be applied which does not use Jacobian process, rather extracts the model characteristics statistically. The UKF is more accurate than the EKF as in this method also the second moment of the non-linear function is held.

One major requirement of the mentioned filters is a noise with zero mean and Gaussian distribution. Under conditions where the Gaussian requirement is not met, or model of the system is inaccurately defined, or the measured data are erroneous, the filter may diverge. Many efforts are invested to improve the stability of the filters in such conditions. One solution is a robust Kalman filter known as H_∞ filter which is designed specifically for robustness. Unlike the Kalman filter which is designed to estimate the mean of the power, the H_∞ filter limits the maximum power of the noise to a worst case condition. In addition to that, the H_∞ filter does not make any assumption about the noise characteristics as these parameters are defined deterministically. This is a very advantages feature as in many applications these parameters are not known in advance and the noise characteristics may also change during the operation of the system [3].

A few researchers have studied the performances of the H_∞ filter and compared it with other filtering approaches. Examples of comparison of H_∞ filter with Kalman filter are provided in [2, 4, 5]. Cao et al. [6] have applied H_∞ filter for blind navigation application. Other papers who address application of H_∞ filter in localization area are [1, 7, 8]. Also modified version of H_∞ filter such as unscented H_∞ filter or adaptive H_∞ filter are proposed in the works [9, 10].

Although the performance of this filter has been addressed before, most of these evaluations are done only theoretically or by simulation. This paper addresses the application of H_∞ filter in time differential localization systems and compares the general performances of this filter in terms of accuracy, runtime speed, convergence speed and resistivity against non-ideal conditions with EKF and UKF in practical experiments.

II. STRUCTURE OF THE H_∞ FILTER

The application of H_∞ filter for non-linear systems is similar to EKF in that Jacobian process is applied for linearization. The result is extended H_∞ filter which is exploited in our project for localization of robots in a differential time of arrival (TDoA) localization system. The model of a localization system in the form of state space can be defined as:

$$x_{k+1}^{-} = Ax_k + w_k \tag{1}$$

$$y_{k+1} = h(x_{k+1}^{-}) + v_k \tag{2}$$

where x_{k+1}^{-} is the *a priori* matrix of the state variables after prediction including the location and speed of the mobile node in each axis, A is the fundamental matrix of the model which defines the dynamics of the node, y is the differential distance measurements of the node, and w and v are the process noise and measurement noise respectively. The non-linear observation function h in TDoA topology is defined according to the following equation:

$$d_{m,n} = \sqrt{(x-x_m)^2 + (y-y_m)^2} - \sqrt{(x-x_n)^2 + (y-y_n)^2} \tag{3}$$

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where (x, y) are coordinates of the mobile node, (x_m, y_m) and (x_n, y_n) are coordinates of the anchors m and n respectively and $d_{m,n}$ is the differential range between the mobile node and the anchors m and n. The first step of the filter, is to linearize the nonlinear function h around the states operating point x_k using Jacobian which is defined as:

$$H_k = \left.\frac{\partial h(x)}{dx}\right|_{x_k} \quad (4)$$

The H_∞ filter defines a bound to the maximum energy of the noise by defining following equation:

$$D_k = (I - \gamma S_k P_k + H_k^T R_k^{-1} H_k P_k)^{-1} \quad (5)$$

where I is the identity matrix, P_k is the error of the estimate, the matrices S_k and R_k are defined by user deterministically and γ is the defined bound which limits the noise and model error worst cases. This parameter is a value close to zero which should be selected carefully according to the application to avoid filter divergence. In the next step, the filter gain is defined as:

$$K_k = A_k P_k D_k H_k^T R_k^{-1} \quad (6)$$

The final steps of the filter are prediction and update which are described below together with the error estimate equation as:

$$\hat{x}_{k+1} = A_k \hat{x}_k + K_k\big(y_k - h(\hat{x}_k)\big) \quad (7)$$

$$P_{k+1} = A_k P_k D_k A_k^T + Q_k \quad (8)$$

where Q is also a matrix defined and acquired empirically by user to avoid covariance matrix P_{k+1} converging to zero.

III. Perfromance Evaluation and Experiments

In the first experiment, the amount of localization noise for each filtering algorithm in ideal conditions is evaluated. For this purpose, a stationary node is used which was located in the middle of a test field with dimensions of $60 \times 40\text{cm}^2$ and surrounded by 4 anchors located at each corner of the field. The standard deviation of the measurements are provided in the form of probability distribution function (PDF) in Fig. 1. The amount of observed noise for these filtering techniques are almost identical with a mean value of 2.66 for EKF, 2.68 for UKF and 2.64 for H_∞ filter. The distribution of the noise in all cases is Rician with mean 2.39 and variance of 1.32.

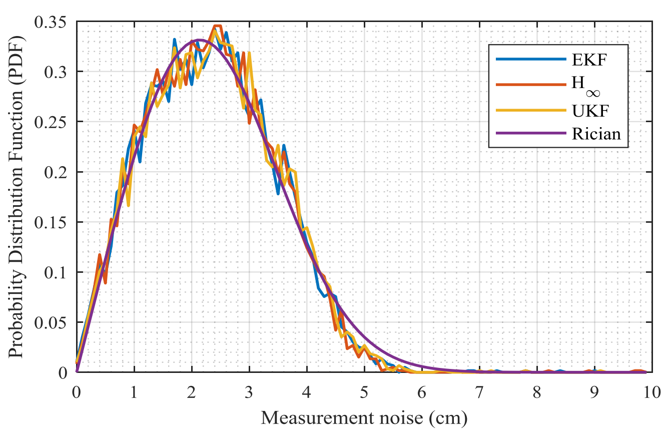

Fig. 1. The standard deviation of the noise in location measurements of different filtering techniques with Rician distribution

In the next experiment, the effect of initial point on the convergence of filters are evaluated. For the case of the fixed node which is used in the previous setup, different initial points are considered. These are (1,1), (80,50), (-10,80), (-1,-5). The real position of the node is (34,20). The selection of the initial points are based on criteria such as distance and different sign of the location to evaluate the behavior of filter in these complex conditions. The results of filter convergence for the mentioned methods and only axis x are provided in Fig. 2. For the case of initial point (1,1) (Fig.2.a) all the filters converge quickly with faster convergence of EKF. This is an ideal condition for filters as the initial values are positive, in the vicinity of the target and smaller than the final value. For the next initial point (80,50), the filters need to decrease the value of axis x. In this case, the UKF has a better performance by converging immediately to close vicinity of the target point. The EKF needs more time and tends to be unstable in this case. These bounces are however minimum for H_∞ filter. For the next point (-10,80), one negative number is provided which results in divergence of UKF. In the last step, both values are negative which again results in divergence of UKF as well as instability of EKF. The superior behavior of H_∞ filter in these cases are clearly visible. As a conclusion, it can be deducted that the H_∞ filter has robust behavior when the initial points are badly selected, however when the points are ideally selected, the performances of other filters could be better.

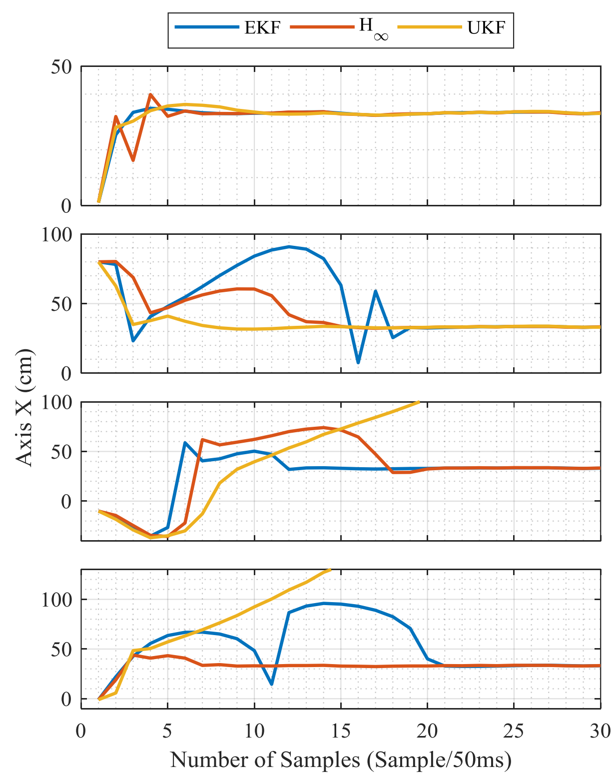

Fig. 2. Convergence speed and behaviour of different filters for different initial points from top to bottom (1,1), (80,50), (-10,80), (-1,-5)

In the next experiment, the effect of erroneous data on the filter's performance is evaluated. For this sake, a barrier is used to block the line of sight between the stationary node used in the setup and the anchor nodes to simulate non-line of sight (NLOS) conditions. These conditions normally create a large bias error

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in the distance measurements which in some cases result in filter divergence. In our experiment, we have created a severe NLOS condition which resulted in large bias error. The results of node localization for the filters are provided in Fig. 3. As it can be seen, this error resulted in divergence of the UKF. Also the EKF is largely deviated from the real point of the node with error of around 20m, but still could manage to return to the real point when the NLOS condition is removed. Unlike the other filters, H_∞ filter resists against the non-ideal and erroneous data measurements and only experiences an error of 2m.

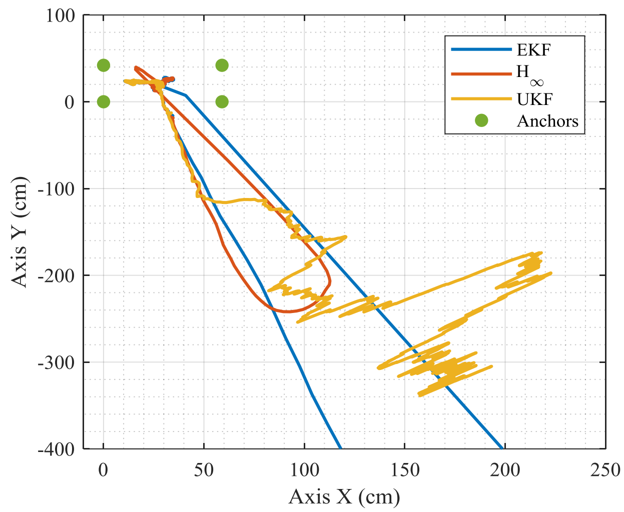

Fig. 3. Localization results of a stationary node with NLOS condition

The localization error of this experiment for different filters are presented in Fig. 4. According to the results, it can be seen that the H_∞ filter mitigates the error of the localization about one sixth of the EKF and avoids divergence of the filter.

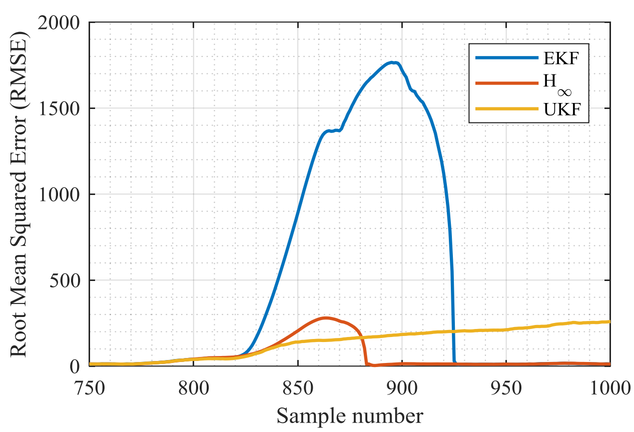

Fig. 4. Localization error of different filter for a node in NLOS condition

The performance of the H_∞ filter in terms of runtime as well as implementation complexity is evaluated. The results are provided in Table I. According to these results, the fastest runtime belongs to EKF. The H_∞ filter is slightly slower than EKF but still is twice faster than the UKF. Due to the structure of the UKF which needs to integrates sigma points and evaluates the noise characteristics at each iteration, the runtime of this algorithm is long and the filter has the highest implementation complexity among the other filters.

TABLE I. COMPARISON OF THE FILTERS' CHARACTERISTICS

Parameter	***EKF***	***UKF***	$\boldsymbol{H_\infty}$
Minimum runtime	20.52 μs	71.84 μs	39.91 μs
Implementation complexity	Low	High	Medium

IV. CONCLUSION

In this paper, the robust H_∞ filter is introduced and its performances against other common filtering methods namely EKF and UKF which are used for localization systems are compared. The filters are evaluated in a series of practical experiments with the focus on analyzing the behavior of the filter in non-ideal and presence of erroneous measurements. The results of experiments indicated that, the H_∞ filter is robust against badly selected initial points but sub-optimal when the conditions are ideal. Also in case of presence of erroneous data such as NLOS conditions, the filter shows resistance against divergence and introduces only a small portion of deviation (1/6) compared to EKF. As a conclusion, it can be stated that, the H_∞ filter proves to be a good choice for challenging conditions of localization systems especially in indoor area where the NLOS condition is highly expected. Although the filter is sub-optimum in ideal conditions, the robustness of the filter is promising with slightly longer runtime but smaller implementation overhead compared to UKF algorithm.

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